RAN-2103000206020033

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T.Y. B.Sc. (Sem. VI) Examination March - 2025

Mathematics (Paper - MTH : 603)

Real Analysis - III

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book
Name of the Examination:
☛ **T.Y. B.Sc. (Sem. VI)**
Name of the Subject :
☛ **Mathematics (Paper - MTH : 603) Real Analysis - III**
Subject Code No.: **2103000206020033**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations.

Q. 1. Answer the following questions (Any Five): 10

1. Prove that a sequence {1,1,1,1,1,1, ...} is (C, 1) summable.
2. Define: (C, 2) summability of an infinite series.
3. Let $f_n(x) = x^n$; $0 \le x \le 1$ Does a sequence $\{f_n\}_{n=1}^{n=\infty}$ converge uniformly on [0,1]? Justify your answer.
4. Define: Point wise convergence sequence of functions.
5. Show that every singleton set is of measure zero.
6. Let $\sigma = \left\{0, \frac{1}{2}, 1\right\}$ be a subdivision of [0,1], then find the refinement of σ.
7. Evaluate: $\lim\limits_{n\to\infty} \frac{1}{n}\left[\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \dots\dots.. + \sin\frac{n\pi}{n}\right]$.
8. If $F(x) = \int_0^x \sqrt{t+t^4}\, dt$ ($x > 0$), then find $f(2)$.

Q. 2. Answer the following (Any two): 10

a. If the sequence of real numbers $\{S_n\}_{n=1}^{n=\infty}$ converges to L, then prove that it is also $(C, 1)$ summable to L.

b. Show that the series $\Sigma_{n=1}^{n=\infty}(-1)^n$ is $(C, 1)$ summable to $\frac{-1}{2}$.

c. Is the sequence 1,–1, 2,–2, 3,–3, 4,–4... (C, 1) summable? Justify your answer.

Q. 3. Answer the following (Any two): 10

a. Let $\{f_n\}_{n=1}^{n=\infty}$ be a sequence of continuous real valued functions in $R[a,b]$; which converges uniformly to the function f on $[a, b]$. Then prove that $f \in R[a, b]$.

b. State and prove Cauchy criterion for uniformly convergence sequence of functions.

c. Let $f_n(x) = \frac{x}{n} e^{\frac{x}{n}}$; $0 \le x < \infty$. Then

(i) Does $\{f_n\}_{n=1}^{n=\infty}$ converge uniformly on [0, 500]? Justify,

(ii) Does $\{f_n\}_{n=1}^{n=\infty}$ converge uniformly on [0, 1]? Justify.

Q. 4. Answer the following (Any two): 10

a. Show that the set {a, b, c, d} is of measure zero.

b. Let $f(x) = x$; $x \in [0, 1]$ and let $\tau = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ be any subdivision of [0,1]. Then compute $U[f; \tau]$ and $L[f; \tau]$.

c. If f is continuous real valued function on $[a, b]$, then prove that $f \in R[a, b]$.

Q. 5. Answer the following (Any two): 10

a. If f is continuous on closed bounded interval $[a, b]$ and if $\varphi'(x) = f(x)$, $x \in [a, b]$, then prove that $\varphi(b) - \varphi(a) = \int_a^b f(x)$.

b. State and prove mean value theorem for integrals.

c. Let f be a continuous real valued function on $[a, b]$ and if

$F(x) = \int_a^x f(t)\,dt$; $x \in [a, b]$, then prove that F is continuous on $[a, b]$.